

ADDITIONAL MATHEMATICS Paper 1 MARK SCHEME Maximum Mark: 80 0606/13 May/June 2016

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Published

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## Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied
WWW	without wrong working

Q	uestion	Answer	Marks	Guidance
1	(i)	-27	<b>B</b> 1	
	(ii)	9 - 8k = 0	M1	for use of discriminant with a complete method to get to $k =$
		$k = \frac{9}{8}$	A1	
		<b>Or</b> $\frac{dy}{dx} = 4x - 3$	M1	for a complete method to get to $k =$
		when $\frac{dy}{dx} = 0$ , $x = \frac{3}{4}$ so $k = \frac{9}{8}$	A1	
		Or completing the square $y = 2\left(x - \frac{3}{4}\right)^2 + k - \frac{9}{8}$	M1	for a complete method to get to $k =$
		$k = \frac{9}{8}$	A1	
2	(a)	$2^{4(3x-1)} = 2^{3(x+2)}$ or $4^{2(3x-1)} = 4^{\frac{3}{2}(x+2)}$ or $8^{\frac{4}{3}(3x-1)} = 8^{x+2}$ or $16^{3x-1} = 16^{\frac{3}{4}(x+2)}$	B1	<b>B1</b> for a correct statement
		or $16^{3x-1} = 164^{x}$ leading to $x = \frac{10}{9}$ cao	M1 A1	for equating indices
	(b)	$p = \frac{5}{3}$ $q = -2$	B1 B1	
		q = -2	DI	

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Question	Answer	Marks	Guidance
3	On <i>x</i> -axis, $2x^2 - 7 = 1$ x = 2	M1 A1	for equating to 1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4x}{2x^2 - 7}$	<b>B</b> 1	
	When $x = 2$ , $\frac{\mathrm{d}y}{\mathrm{d}x} = 8$		
	Gradient of normal = $-\frac{1}{8}$		
	Equation of normal $y = -\frac{1}{8}(x-2)$	M1	for attempt at perpendicular through <i>their</i> $(2, 0)$ , must be using $y = 0$
	Required form $x + 8y - 2 = 0$	A1	must be equated to zero with integer coefficients
4 (a)	$\mathbf{A}^2 = \begin{pmatrix} 7 & -2 \\ -3 & 6 \end{pmatrix}$	B1	
	$\mathbf{A}^2 - 2\mathbf{B} = \begin{pmatrix} 1 & -2 \\ -5 & 2 \end{pmatrix}$	M1 A1	for their $\mathbf{A}^2 - 2\mathbf{B}$
(b)	$ \begin{pmatrix} 4 & 1 \\ 10 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} $		
	$so\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 3 & -1 \\ -10 & 4 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ leading to $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$	M1 DM1	for pre-multiplication by <i>their</i> inverse matrix <b>DM1</b> for attempt at matrix multiplication
	(y)  (-3) $x = 1$ $y = -3$	A1 A1	Allow in matrix form
5 (i)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{e}^{4x}}{4} - x\mathrm{e}^{4x}\right) = \mathrm{e}^{4x} - \left(\left(x \times 4\mathrm{e}^{4x}\right) + \mathrm{e}^{4x}\right)$	B1	for $\frac{d}{dx}\left(\frac{e^{4x}}{4}\right) = e^{4x}$
	$=-4xe^{4x}$	M1 A1 A1	for attempt to differentiate a product for a correct product for correct final answer
(ii)	$\int_{0}^{\ln 2} x e^{4x} dx = -\frac{1}{4} \left[ \frac{e^{4x}}{4} - x e^{4x} \right]_{0}^{\ln 2}$	B1FT	<b>FT</b> for use of <i>their</i> $\frac{1}{p} \times \left(\frac{e^{4x}}{4} - xe^{4x}\right)$ , must
	$= -\frac{1}{4} \left( \left( \frac{16}{4} - 16\ln 2 \right) - \frac{1}{4} \right)$	B1 M1	be numerical p, but $\neq 0$ for $e^{4\ln 2} = 16$ for correct use of limits, must be an integral of the correct form
	$=4\ln 2 - \frac{15}{16}$	A1	

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G	uestion	Answer	Marks	Guidance
6	(i)	$2-\sqrt{5} < f(x) \leq 2$	B2	<b>B1</b> for $\leq 2$ <b>B1</b> for $2 - \sqrt{5} <$ or awrt $-0.24$ Must be using f, f(x) or y, $2 - \sqrt{5} <$ , if not then <b>B1</b> max
	(ii)	$f^{-1}(x) = (2-x)^2 - 5$ Domain $2 - \sqrt{5} < x \le 2$ Range y or $-5 \le f^{-1}(x) < 0$	M1 A1 B1 B1	for a correct method to find the inverse Must be using the correct variables for the B marks
	(iii)	$fg(x) = f\left(\frac{4}{x}\right)$ $= 2 - \sqrt{\frac{4}{x} + 5}$ leading to $x = -4$	M1 DM1 A1	for correct order of functions for solution of equation
7	(i)	Finding an angle of $68.2^{\circ}$ or $21.8^{\circ}$ $\frac{4.5}{\sin 68.2} = \frac{2.4}{\sin \alpha}$ leading to $\alpha = 29.7^{\circ}$ (allow ±0.1) Direction is 82.1° to the bank, upstream(allow ±0.1°)	B1 B1 B1 B1 B1	for the sine rule
	(ii)	$\frac{4.5}{\sin 68.2} = \frac{2.4}{\sin 29.7} = \frac{v_r}{\sin 82.1}$ leading to $v_r = 4.8$ time taken = $\frac{80.78}{4.8} = 16.8$ Alternative method:	B1 B1 M1 A1	for the sine rule for resultant velocity for attempt to find <i>AB</i> and hence the time taken
		Finding an angle of $68.2^{\circ}$ or $21.8^{\circ}$ $4.5^2 = 2.4^2 + v_r^2 - (2 \times 2.4 \times v_r \cos 68.2)$ leading to $v_r = 4.8$ Use of sine rule to obtain angle and direction to obtain direction is $82.1^{\circ}$ to the	B1 B1 B1 B1 B1 B1	for correct use of the cosine rule for resultant velocity for use of the sine rule for $\alpha = 29.7^{\circ}$
		bank, upstream Use of time taken = $\frac{80.78}{4.8} = 16.8$	B1 M1 A1	for 82.1° for attempt to find <i>AB</i> and hence the time taken

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Qı	uestion	Answer	Marks	Guidance
8		$y-6 = -\frac{4}{12}(x+8)$ (3y+x=10)	M1 A1	for a correct method allow unsimplified
	(ii)	y-7=3(x+1) (y=3x+10)	DM1 A1	for attempt at a perpendicular line using $(-1, 7)$ allow unsimplified
	(iii)	point of intersection $(-2, 4)$ which is the midpoint of <i>AB</i>	M1 M1 A1	for attempt to find the point of intersection using simultaneous equations for attempt to find midpoint for all correct
		Alternative method: Midpoint $(-2, 4)$ Verification that this point lies on <i>CP</i> .	M1 M1 A1	for attempt to find midpoint for full verification for all correct
	(iv)	$CP = \sqrt{10} \text{ or } 3.16$	<b>B</b> 1	
	(v)	Area = $\frac{1}{2} \times \sqrt{10} \times 4\sqrt{10}$	M1	for correct method <b>using</b> <i>CP</i>
		= 20	A1	for 19.9 – 20.1

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Q	uestion	Answer	Marks	Guidance
9	(i)	$2\cos x \cot x = \cot x + 2\cos x$ $2\cos x \frac{\cos x}{\sin x} + 1 = \frac{\cos x}{\sin x} + 2\cos x$	M1	for use of $\cot x = \frac{\cos x}{\sin x}$ for both terms
		$2\cos^2 x + \sin x = \cos x + 2\cos x \sin x$	DM1	for multiplication throughout by $\sin x$
		$2\cos^2 x - 2\cos x \sin x = \cos x - \sin x$		
		$2\cos x(\cos x - \sin x) = \cos x - \sin x$	DM1	for attempt to factorise
		$(2\cos x - 1)(\cos x - \sin x) = 0$	A1	for completely correct solution www
		Alternative method:		
		$a\cos^{2} x - a\cos x\sin x - b\cos x$ $+ b\sin x = 0$	M1	for expansion of RHS
		$a\cos x \cot x - a\cos x - b\cot x + b = 0$	DM1 DM1	for division by $\sin x$ for comparing like terms to obtain both <i>a</i> and <i>b</i>
		a = 2,  b = 1	A1	for both correct www
	(ii)	$(2\cos x - 1)(\cos x - \sin x) = 0$		
		$\cos x = \frac{1}{2} , \tan x = 1$	M1	for either
		$x = \frac{\pi}{3} , x = \frac{\pi}{4}$	A1,A1	A1 for each, penalise extra solutions within the range by withholding the last A mark
		Alternative method: $(2\cos x - 1)(\cot x - 1) = 0$		
		Leading to $\cos x = \frac{1}{2}$ , $\tan x = 1$	M1	for attempt to factorise the original equation and attempt to solve
		$x = \frac{\pi}{3}, x = \frac{\pi}{4}$	A1,A1	A1 for each, penalise extra solutions within the range by withholding the last A mark
10	(i)	f(-2) = -32 - 2k + p = 0	M1	for attempt at $f(-2)$
		$f'\left(\frac{1}{2}\right) = \frac{12}{4} + k = 0$	M1	for attempt at $f'\left(\frac{1}{2}\right)$
		leading to $k = -3$ and $p = 26$	A1,A1	A1 for each
	(**)			
	(ii)		B1FT	<b>FT</b> for <i>their</i> $\frac{p}{2}$
		$(x+2)(4x^2-8x+13)$	B1	all correct
	(iii)	Showing that $4x^2 - 8x + 13 = 0$ has no real roots	M1,	M1 for a valid attempt at solution of equation leading to no solution or
		so $x = -2$ only www	A1	consideration of the discriminant

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Question	Answer	Marks	Guidance
11 (i)	$AB = 2r\sin\theta$ or $\sqrt{r^2 + r^2 - 2r^2\cos 2\theta}$	B1	
	or $\frac{r\sin 2\theta}{\sin\left(\frac{\pi}{2} - \theta\right)}$		
	or $\frac{r\sin 2\theta}{\cos \theta}$		
(ii)	$2r\sin\theta + 2r\theta = 20$	M1	for use of (i) + arc length = 20, oe
	$r = \frac{10}{\theta + \sin \theta}$	A1	must be convinced
(iii)	$\frac{\mathrm{d}r}{\mathrm{d}\theta} = -\frac{10(1+\cos\theta)}{(\theta+\sin\theta)^2}$	M1 A2,1,0	for a correct attempt to differentiate -1 each error
	When $\theta = \frac{\pi}{6}$ , $\frac{\mathrm{d}r}{\mathrm{d}\theta} = -17.8$	A1	allow awrt –17.8
(iv)	$\frac{\mathrm{d}r}{\mathrm{d}t} = 15$	B1	may be implied
	$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{\mathrm{d}r}{\mathrm{d}t} \div \frac{\mathrm{d}r}{\mathrm{d}\theta}$	M1	for use of $\frac{15}{their}$ (iii)
	$\frac{\mathrm{d}\theta}{\mathrm{d}t} = -0.842$	A1	allow -0.84 or -0.843